Experiment No: M9

Experiment Name: Simple Pendulum

Objective: Investigating the relation between length and period of a simple pendulum. Measuring the gravitational acceleration using a simple pendulum.

Keywords: Simple pendulum, period, simple harmonic motion, gravitational acceleration.

Theoretical Information:

A body tied by a rope and hung from a stationary point (also called pivot) is called a simple pendulum.

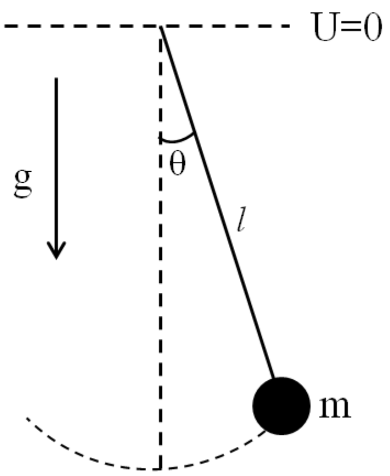

Figure 9.1 Simple Pendulum

Equation of motion for a simple pendulum can be derived from several different approaches like using force, torque and energy arguments. Here we will use the energy approach. We will assume the gravitational potential energy (U) at the height of the pivot as zero. Then we can write the total energy of the simple pendulum as follows.

$$\frac{1}{2}mv^2 - mglcos\theta = E_{total} \qquad 9.1$$

While the first term on the left hand side of the equation 9.1 represents the kinetic energy of the pendulum second term stands for the potential energy. The speed of the pendulum can be written in terms of its angular velocity with respect to the pivot as $v = l\frac{d\theta}{dt}$ If we substitute this in to the equation 9.1 we get the following expression:

$$\frac{1}{2}ml^2\left(\frac{d\theta}{dt}\right)^2 - mglcos\theta = E_{total} \qquad 9.2$$

We know from the principle of energy conservation that the total energy of the system should stay constant. That means it will not be dependent on time. So if we take the derivative of both sides of equation 9.2 with respect to time it will give us the following expression where the right hand side simply vanishes.

$$ml^2\left(\frac{d\theta}{dt}\right)\left(\frac{d^2\theta}{dt^2}\right)+mgl\left(\frac{d\theta}{dt}\right)sin\theta=0 \qquad 9.3$$

For further simplification the term $ml^2\left(\frac{d\theta}{dt}\right)$ can be cancelled out from the left hand side since they are all different from zero. In this case the equation of motion for the pendulum can be written as follows:

$$\frac{d^2\theta}{dt^2}+\frac{g}{l}sin\theta=0 \qquad 9.4$$

Unfortunately equation 9.4 is a "non-linear" differential equation and it doesn't have an easy solution. However if we assume that the oscillations will be "small enough" to be able to use the approximation $sin\theta \approx \theta$ conveniently, the equation can be rewritten as follows:

$$\frac{d^2\theta}{dt^2}+\frac{g}{l}\theta=0 \qquad 9.5$$

Equation 9.5 is known as the simple harmonic motion equation. It can easily be demonstrated that the following solution satisfies the equation 9.5.

$$\theta = A.\cos(\omega t + \varphi) \qquad 9.6$$

As it is seen the system makes a periodic motion. Here while A ve $\emptyset$ are two arbitrary constants which only depend on the initial conditions of the motion ω is called the angular frequency of the system and it is given by the expression:

$$\omega=\sqrt{\frac{g}{l}} \qquad 9.7$$

By definition the relation between the angular frequency and the period of the system is:

$$T=\frac{2\pi}{\omega} \qquad 9.8$$

If we put equation 9.7 into equation 9.8 we get the following expression for the period of the system.

$$T_0=2\pi\sqrt{\frac{l}{g}} \qquad 9.9$$